The Socratic Series

Physics II

Higher-Order Questions & Support Materials

Paper No. 2/2016

THE INSTITUTION
for Science Advancement

About the Institution

As Malaysia is heading into a more competitive era of innovation led economic growth, there have been much to say on it preparation to meet this new challenge.

According to a 2012 study* by the Academy of Sciences Malaysia ("ASM") (an agency under the Ministry of Science, Technology and Innovation, "MOSTI"), there is an alarming shortfall of students and professionals involved in the fields of science, technology, engineering and mathematics ("STEM").

The study shows that less than 30% of secondary schools students are enrolled in the Science Stream. Of this, an even smaller proportion elects to pursue STEM degrees in tertiary education, and an even fewer ends up in science and technology-based professions.

It is estimated that by the year 2020, the nation requires around 500,000 STEM professionals from lab technicians to full-time researchers in all fields of the natural sciences, both in academia and industry.

Thus far Malaysia's stock of STEM professionals is only around 1/10th of that figure, with only a few years to the 2020 deadline.

The study had identified that this shortfall is due to the unpopularity of science subjects, which are purported to be difficult to master and less interesting than the humanities. This is compounded by the students' limited awareness on real world STEM professions such as chemical engineering, bioinformatics, applied mathematics and agronomics.

With this in mind, the Institution for Science Advancement (the "Institution") was envisioned as an independent, apolitical non-government organisation ("NGO"). The Institution's goal is to address the nation's underdeveloped scientific community through projects which encompass secondary and tertiary levels of education as well as postgraduate and professional levels of academia.

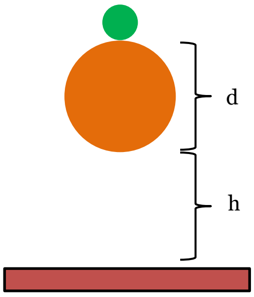
About the Socratic Series

The Socratic Series is to serve as a supplement to the science and mathematics syllabus. The questions and materials presented assesses a student's understanding of concepts from a subject on the tertiary preparatory level, as well as reasoning skills derived from lab experience.

The questions, as all questions should be, are to be thought provoking and not simply a formulaic fill-in-the-blanks queries, but involves lessons in problem-solving and critical thinking.

Questions (Answers on Page 4)

1. A tennis ball with (small) mass m2 sits on top of a basketball with (large) mass m1. The bottom of the basketball is a height h above the ground, and the bottom of the tennis ball is a height h + d above the ground. The balls are dropped. To what height does the tennis ball bounce?



2. **Passing the Spaghetti**

   At a dinner party, there are N people seated around a table. A plate of spaghetti starts at the head of the table. The person sitting there takes some spaghetti and then passes the (very large) plate at random to his/her right or left. Henceforth each person receiving the plate takes some spaghetti and then passes the plate at random to his/her right or left. (Diners who have already received the plate can simply pass it on, without taking any more.) When all the diners have finally received their spaghetti, the plate stops being passed, and the eating begins.

   What are the chances of being the last to be served, as a function of position (relative to the head) at the table of N people?

3. **Green Eyed Dragons**

   You visit a remote desert island inhabited by one hundred very friendly dragons, all of whom have green eyes. They haven't seen a human for many centuries and are very excited about your visit. They show you around their island and tell you all about their dragon way of life (dragons can talk, of course).

   They seem to be quite normal, as far as dragons go, but then you find out something rather odd. They have a rule on the island which states that if a dragon ever finds out that he/she has green eyes, then at precisely midnight on the day of this discovery, he/she must relinquish all dragon powers and transform into a sparrow. However, there are no mirrors on the island, and they never talk about eye color, so the dragons have been living in blissful ignorance throughout the ages.

   Upon your departure, all the dragons get together to see you off, and in a tearful farewell you thank them for being such hospitable dragons. Then you decide to tell them something that they all already know (for each can see the colors of the eyes of the other dragons). You tell them all that at least one of them has green eyes. Then you leave, not thinking of the consequences (if any). Assuming that the dragons are (of course) infallibly logical, what happens?

   If something interesting does happen, what exactly is the new information that you gave the dragons?

4. **Great Balls of Fire**

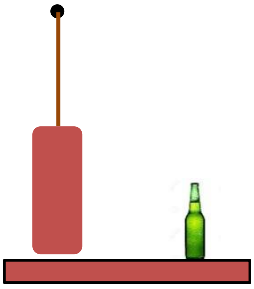
   You are given two identical steel balls of radius 5 cm. One ball is resting on a table; the other ball is hanging from a string. Both balls are heated until their radii have increased to the same value of 5.01 cm. Which ball absorbed more heat and why?

5. You are given two spheres that are identical in size, weight, appearance, and touch but one sphere is hollow while the other is solid. Using only simple items that you might find at home (no drills, no X-ray machines), determine which sphere is hollow.

6. **Let Go or Hang On?**

   A painter is high up on a ladder painting a house when, unfortunately, the ladder starts to lean over and fall. Determine which is the less harmful action for the painter: to let go of the ladder right away and fall to the ground, or to hang on to the ladder all the way to the ground.

7. A heavy 300 kg sandbag one-meter-tall is hung from a playground swing with a rope 3 meters long so that the bottom of the sandbag just clears the ground. A bottle is then placed on the ground a meter away from the sandbag as shown.



Explain how to knock the bottle over with the sandbag if you are given a paper drinking straw but are not allowed to touch anything (sandbag, rope, bottle, swing) with your body or with the straw.

8. **Two Balloons**

Consider two identical spherical birthday balloons, one of which is inflated to 2/3 its maximum diameter and the other inflated to 1/3 its maximum diameter. What happens when the openings of the two balloons are connected to each other by a straw so that air can flow back and forth between the two balloons?

9. **Ice and Water**

A kilogram of ice at 0 Celsius and a kilogram (liter) of boiling water at 100 Celsius are mixed together in a thermally insulated tank. Will all the ice melt?

10. **Clouds**

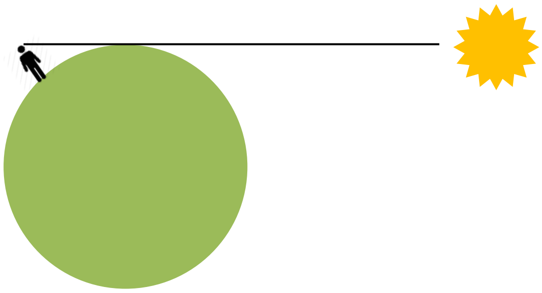
As someone living near the beginning of the 21st century, can you explain a problem that badly perplexed the ancient Greeks and Romans (and also people throughout the medieval ages): how come clouds don't come crashing down to the ground? After all, clouds are made of water droplets and ice crystals which are about 800 times denser than air, comparable in density to rocks. So why don't clouds fall like a rock to the ground?

11. **Sunset Math**

You are enjoying a vacation and happen to have a stopwatch with you at the beach. As you watch the sun set over the ocean, you carry out the following eccentric sequence of events:

(1), you lie down on your stomach in the sand and wait until the top of the sun just disappears below the horizon; (2), you then quickly stand up and simultaneously start your stopwatch. By standing up, a bit of the sun is now visible again; and (3), you wait until the top of the sun again dips below the horizon, at which point you stop the stopwatch.

Knowing this elapsed time, your height, and that a day lasts 24 hours, explain how you can deduce the radius of the Earth.

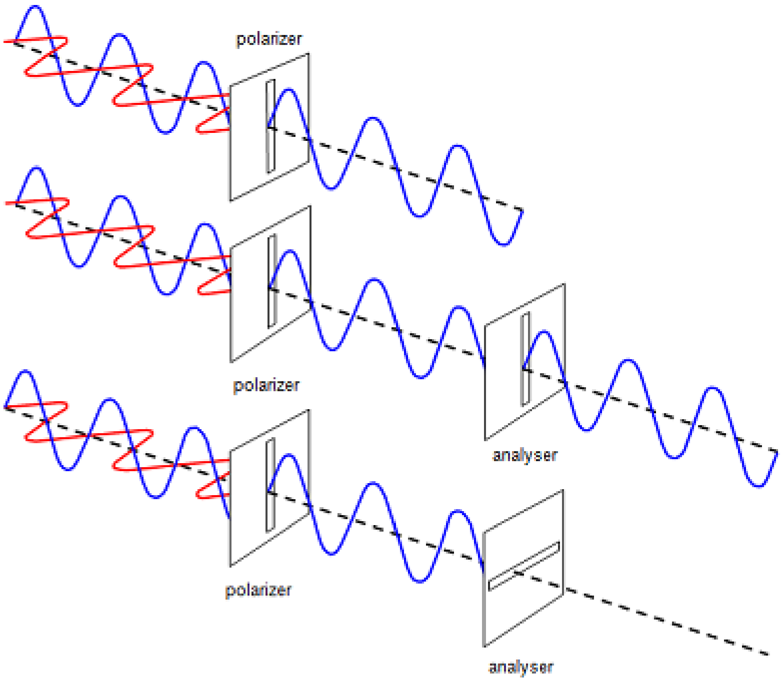


12. **The Weight Loss Space Program**

Assuming that the earth is spherical with radius R = 6400 km and taking into account that it rotates once per day, calculate how much less you weigh at the earth's equator than if you were standing at the north or south pole.

13. Polarizing Paradox

Many sunglasses have the property of being "polaroid" which means that they filter the light in a special way (they produce what is called "linearly polarized light"). Now a single pair of polaroid sunglasses is a strong filter and substantially reduces the intensity of light passing through it. If you look through two pairs of polaroid sunglasses simultaneously, you get a further reduction in light intensity but something funny happens: if you rotate the second pair of sunglasses by 90° while holding the first fixed, you can reduce the amount of light that gets through almost to nothing; one sees nearly total darkness. As illustrated:



So here is something strange to think about: if you hold two pairs of sunglasses so that almost no light gets through, and then you insert a third pair of polaroid sunglasses between the first two pairs, you will now find that, for certain orientations of the middle (third) pair of sunglasses, light gets through again. Explain this: how is it possible that one can add a filter that, by itself, reduces light intensity but here increases the light intensity? Can you identify and explain the angle at which you should hold the middle pair of sunglasses to maximize the amount of light that gets through?

14. **Submarines vs Balloons**

Explain why an inflated balloon (made of a rigid plastic material) will rise to a definite height once it starts to rise, while a submarine will always sink to the bottom of the ocean once it starts to sink.

15. **The Inverse Rocket**

Consider a cylindrical can of highly compressed gas in outer space. You know that if you puncture a hole in the can so that the gas can leak out, the can will start moving like a rocket, in a direction opposite to that of the leaking gas.

Now consider an "inverse" problem in which a cylindrical can is completely empty (has a vacuum) and is inserted into a big tub of water. Also imagine the experiment being done on the space shuttle so that there is no buoyancy force that would push the can to the surface of the tub. The can is now punctured at one end so that a jet of water starts to stream into the can. In what direction will the can move and why?

Answers

1. For simplicity, assume that the balls are separated by a very small distance, so that the relevant bounces happen a short time apart. This assumption isn't necessary, but it makes for a slightly cleaner solution. Just before the basketball hits the ground, both balls are moving downward with speed (using mv2/2 = mgh)

$$v = \sqrt{2gh}.$$

Just after the basketball bounces off the ground, it moves upward with speed v, while the tennis ball still moves downward with speed v. The relative speed is therefore 2v. After the balls bounce off each other, the relative speed is still 2v. (This is clear if you look at things in the frame of the basketball, which is essentially a brick wall.1) Since the upward speed of the basketball essentially stays equal to v, the upward speed of the tennis ball is 2v + v = 3v. By conservation of energy, it will therefore rise to a height of H = d + (3v) 2/(2g). But v 2 = 2gh, so we have

$$H = d + 9h.$$

2. For the case of n = 3, it is obvious that the two people not at the head of the table have equal 1/2 chances of being the last served (BTLS).

For the case of n = 4, label the diners as A,B,C,D (with A being the head), and consider D's probability of BTLS. The various paths of spaghetti that allow D to be the last served are:

ABC..., ABABC..., ABABABC..., etc.

The sum of the probabilities of these is

$$\frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots = \frac{1/4}{1 - 1/4} = \frac{1}{3}.$$

By symmetry, B also has a 1/3 chance of BLTS, and then that leaves a 1/3 chance for C. Hence, B, C, and D all have equal 1/3 chances of BLTS. The probabilities for n = 5 are a bit tedious to calculate in this same manner, so at this point we will (for lack of a better option) make the following guess: Claim: For arbitrary n, all diners not at the head of the table have equal 1/(n − 1) chances of being the last served (BTLS). This seems a bit counterintuitive (because you might think that the diners further from the head have a greater chance of BTLS), but it is in fact correct.

3. Let's start with a smaller number of dragons, N, instead of one hundred, to get a feel for the problem.

If N = 1, and you tell this dragon that at least one of the dragons has green eyes, then you are simply telling him that he has green eyes, so he must turn into a sparrow at midnight.

If N = 2, let the dragons be called A and B. After your announcement that at least one of them has green eyes, A will think to himself, "If I do not have green eyes, then B can see that I don't, so B will conclude that she must have green eyes. She will therefore turn into a sparrow on the first midnight." Therefore, if B does not turn into a sparrow on the first midnight, then on the following day A will conclude that he himself must have green eyes, and so he will turn into a sparrow on the second midnight. The same thought process will occur for B, so they will both turn into sparrows on the second midnight.

If N = 3, let the dragons be called A, B, and C. After your announcement, C will think to himself, "If I do not have green eyes, then A and B can see that I don't, so as far as they are concerned, they can use the reasoning for the N = 2 situation, in which case they will both turn into sparrows on the second midnight." Therefore, if A and B do not turn into sparrows on the second midnight, then on the third day C will conclude that he himself must have green eyes, and so he will turn into a sparrow on the third midnight. The same thought process will occur for A and B, so they will all turn into sparrows on the third midnight. The pattern now seems clear.

4. The ball resting on table will require more heat.

   When radius of ball increases, the center of mass of the ball resting on table moves up against gravity hence its potential energy also increases. This suggest that this ball requires energy for - (a) to increase its radius due to thermal expansion & (b) to increase its potential energy.

   Whereas the potential energy of the ball hanging from string will decrease. Hence it requires less energy.

5. Roll them down an inclined plane. The hollow one is slower as it has a higher rotational inertia.

6. The painter should let go.

   If he stays on, he will have both angular and linear momentum (angular from the ladder leaning towards the ground, linear from gravity). However, if he lets go, he will only have linear momentum. Thus, he will have a lower velocity when he hits the ground (even if only slightly), and the force exerted on him by hitting the ground will be less.

7. Blowing through the straw to swing the bag is the correct answer. The problem is to demonstrate the strength of resonance. In particular, it should be noted that the formula for the resonant frequency is independent of any mass at the end of a pendulum (the sandbag is just a red herring).

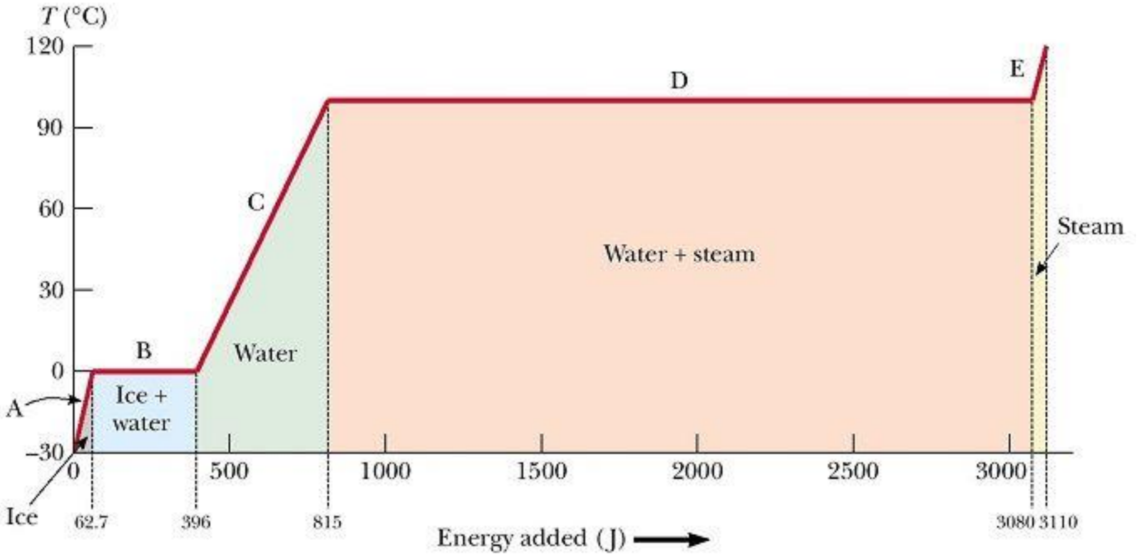
$$T = 2\pi\sqrt{\frac{L}{g}}$$

8. Assuming the balloons are made of elastic material, the air pressure in both should equalize.

9. No. To melt 1kg of ice, we would require more than 1kg of hot water due to principles of specific heat and latent heat.

   The specific heat of a substance is the amount of heat required to change the temperature of 1 gram of the substance by one degree Celsius. By definition, the Specific Heat of water is 1.0, and this amount of heat is called a calorie. A kilocalorie is 1000 calories, also called a "Calorie" with upper case. The Calories in nutrition labels are kilocalories.

   Latent heat is the heat required to change the state of a substance from solid to liquid, or from liquid to gas. Water is in thermal equilibrium with ice when it freezes at the same rate that the ice melts. The heat for melting or freezing water, also called the Heat of Fusion, is 80 calories per gram at 0°C, which is the equilibrium temperature. In other words, it is necessary to add 80 calories of energy to melt one gram of ice at 0°C. Conversely, it is necessary to remove 80 calories of energy to freeze one gram of water at 0°C. The following graph summarizes the effect of energy on the temperature of water in its solid, liquid, and gaseous phases.



10. The two biggest reasons that clouds stay in the sky are 1) small drops, and 2) wind. Small drops of water fall more slowly than big drops. The reason is that as drops fall through the air, the air pushes back on them. Because small drops have less mass and more surface area than large drops, they have a harder time pushing the air out of the way. The kind of wind that most of us are most familiar with is the kind that blows "side-to-side" along the ground. However, the wind sometimes blows up, away from the ground. These updrafts, as they are called, can suspend small drops, preventing them from falling down.

11. Assuming you are 1.7m tall, and it takes x seconds for the sun to disappear into the horizon at step (3). We would deduce that the Earth takes x seconds to spin 1.7m around its axis.

    From this, we extrapolate the x seconds to 24 hours (the time it takes for the Earth to complete its daily spin) to get the circumference of the Earth (assuming this experiment is done at the equator, and not the arctic circle!). Using the circumference, we can use basic geometry to find the radius of the Earth.

12. We can calculate the speed of which an object at the equator travels by first calculating the circumference of the Earth using simple geometry ($C = 2\pi r$). Knowing that this object travels C kilometers every 24 hours, we can derive the speed in which it travels $C/24$ km/h.

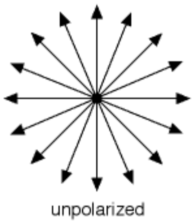
    Using the formula for centripetal acceleration ($F = mass\ x\ velocity^2/radius$) we will then calculate the force that acts on a mass at the equator ($0.03 m/s^2$). Since the Earth's gravitational acceleration is known ($9.807 m/s^2$), we can then find how much lighter an object at the equator is (by 0.31%, so if you weigh 100kg at the north pole, you'd weigh 'only' 99.69kg at the equator.)

13. The reason for this paradox is because of the misapplication of the word "filter." A filter is commonly understood to mean a device that knocks some items out of a stream, while leaving others essentially untouched. A good example of a filter is a sieve — it blocks objects of a particular size, while allowing objects of other sizes to pass through. Another example would be a color filter that knocks out some frequencies of light while letting others through.

    Understood this way, the results of the polarizer experiment are indeed paradoxical. If the all-blocking filters are constructed using sieves or color frequency filters, we are certainly confident that the addition of more filters in the middle of the sequence will not yield different results at the end.

    But what if our so-called "filters" could not only block components of the stream, but also change them? Then we would not be surprised at all if the addition of new "filters" in the middle caused items to get through to the end. If a sieve could not only block particles, but also change their size, or if a color filter could not only block frequencies, but shift light to a different frequency, then all bets are off.

    This, in fact, is what a polarizer does.



unpolarized

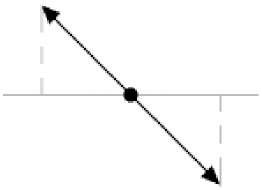
polarized to 0°

Look at the figure above and ask yourself this question: What percentage of the unpolarized light is oriented to exactly 0° or very nearly 0°? Almost none of it — certainly less than 1%. So, if a polarizer simply knocked out undesirable orientations, the strength of the remaining light would be almost entirely gone — it would be less than 1% as strong as the original light source. But you know a polarizer doesn't do that, because you can pick up an ordinary pair of polarizing sunglasses and observe that they're not even particularly dark! Obviously, something else must be happening. Let's see what happens to each of the orientations represented in our simple diagram.

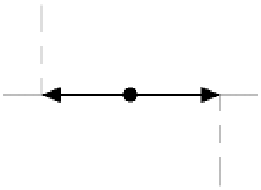
already 0°

polarized to 0°
(unchanged)

We see that light already at 0° is unchanged. We knew that already.

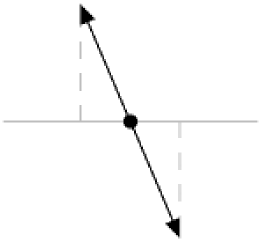


45°

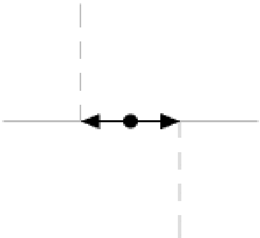


polarized to 0°
(magnitude reduced)

We see what happens to light oriented at 45° — it has its transverse (vertical) component destroyed, and becomes oriented to 0°, but with a weaker magnitude. Simple geometry tells us that it must have a magnitude about 71% of what it had before being polarized to 0° — i.e., 1/sqrt(2) = .707

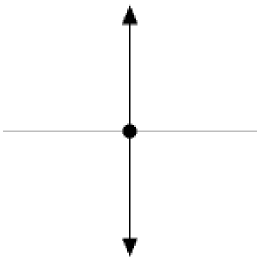


67.5°

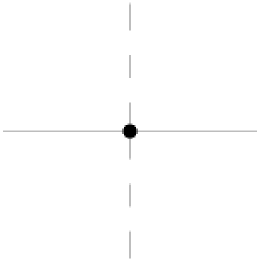


polarized to 0°
(magnitude greatly reduced)

Light that is close to 90° off of the polarizer loses most of its magnitude when being crushed to 0°.



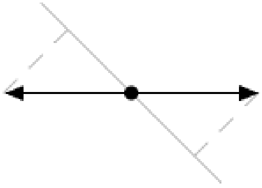
90°



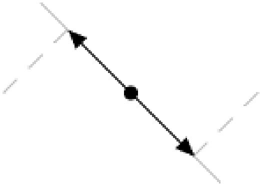
polarized to 0°
(magnitude reduced to zero)

At 90° to the filter being crushed completely out of existence. Figure 12, in fact, illustrates what is happening in Figure 2 when we had only two polarizers in the path of our light source.

With our new understanding of what a polarizer does, it is easy to figure it out the puzzle:

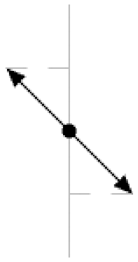


before middle filter

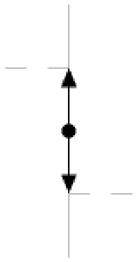


after middle filter

The figure above shows the middle filter taking 0° polarized light (from the first filter) and crushing it to a 45° orientation. This causes the light to drop to about 71% of its magnitude coming from the first filter.



before last filter



after last filter

The last filter taking the 45° polarized light (from the middle filter) and crushing it to a 90° orientation. This causes another 29% drop (0.71x) in magnitude, for an overall drop of exactly 50% (as compared to the results of the single-filter setup). These results can be verified by performing the experiment with an actual light meter

14. The rigid balloon limits the ability of the gas inside to expand, therefore maintaining the density it has at sea level. This does not allow the lifting gas (let's assume its hydrogen) to ascend beyond a certain limit. Submarines are, on the other hand, denser than water at any level of the ocean, and therefore readily sinks to the bottom.

    Although it should be noted that there are such things as 'brine pools'. These pools are bodies of water that have a salinity three to five times greater than the surrounding ocean. Brine pools are sometimes called seafloor "lakes" because the dense brine does not easily mix with overlying seawater. The high salinity raises the density of the brine, which creates a distinct surface and shoreline for the pool. When submarines dive into brine pools, they float on the brine surface due to the high density.

15. The inverse rocket wouldn't move. The principle behind rocketry is Newton's third law of motion: for every action, there is an equal and opposite reaction.

    In a normal rocket engine, the exhaust shoots and pushes itself behind the rocket, and at the same time pushes the rocket forward. The isolation between the force exerted by the exhaust versus the rocket itself makes this possible.

    In an inverse rocket, the water is sucked into the contained vacuum. Intuitively it seems to work on the same principle, but the question now is whether the propulsive force (water jetting into the vacuum) is isolated from the rocket? In the inverse case, it is not, as the inverse rocket is pulling itself forward, the water jet is pushing the rocket back, cancelling out the forces at work in Newton's third law.

    Another way to see it is to imagine a sailing ship. If a gust of wind from the atmosphere were to push on the sails, it would push the ship as well. But what if the wind were to be generated from the ship itself? Imagine a giant fan bolted to the ship deck and blowing into the sail, would the ship move? No, because as the fan blows forward into the sail (action), it also pushes back onto itself (reaction) – cancelling out the forces at work.